APPLICATION OF LAGRANGIAN MODELLING IN URBAN AREAS

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SPECIAL THANKS

- Thanks to Jesse Thé, Lakes Environmental, and Yann Contratto, Olfactoexpert for their continual support

Lakes Environmental

olfactoexpert inc.

PLAN

- Introduction
- Wind field
- Particle model
- Examples

BASE REFERENCE

- H.C. RODEAN, 1996
- ***STOCHASTIC LAGANGIAN MODELS OF TURBULENT DIFFUSION***
- American Meteorological Society, Meteorological Monograph , Volume 26
- <u>JD Wilson Alberta University works are significative</u>

Introduction

- Basic motivation:
  - how to calculate and illustrate in a simple way the wind field around an industrial complex to help in some occasions to refine results of AERMOD
  - and show how a plume could behave
- Everything done here is based on published litterature

Introduction

- 2 blocs are necessary:
  - obtain wind field solution in built areas – industrial complex or urban center
  - resolve the equations for lagrangian transport of parcels

WIND FIELD

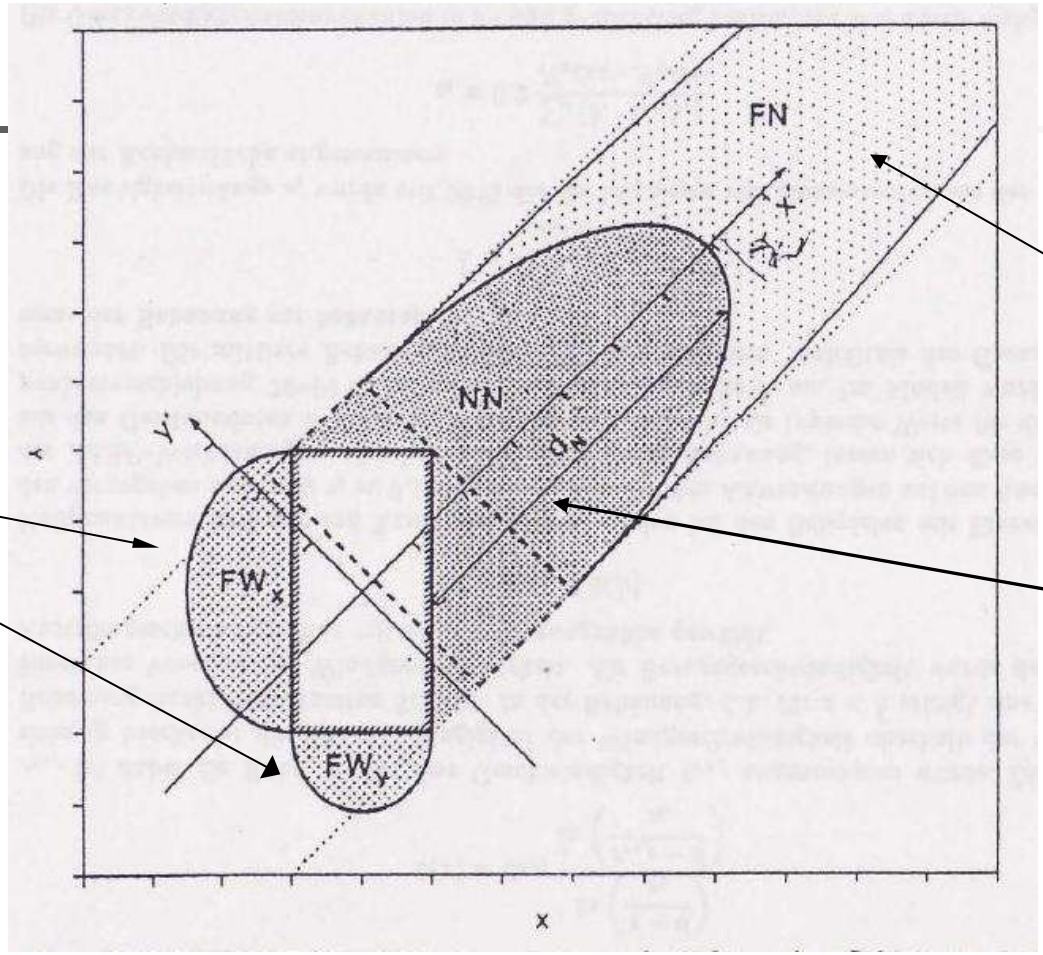
- Options
  - CFD model: solve basic movement equations
    - interesting, precise
    - longer execution time
  - parameter model
    - simplified building effects
    - quite fast

AIRFLO MODEL

- Based on Rockle (1990), Kaplan et Dinar (1996), Los Alamos (2003 and others) following Hosker (1984)

- Wind field parametrized according to influence zone around a building
  - base on one building not too excentric form (cubic or rectangle)

parametrized zones



wake

upfront cavities

rear cavity

upfront cavities

¤	FX¤	FY¤
a_x¤	¶ $L_f \sin^2\theta\sqrt{1-\left(\frac{z}{0.6H}\right)^2}$ ¶ ¤	¶ $\frac{L}{2}$ ¤
a_y¤	¶ $\frac{W}{2}$ ¶ ¤	¶ $L_f \cos^2\theta\sqrt{1-\left(\frac{z}{0.6H}\right)^2}$ ¤
vent· initial¤	$u_0=0$¤	$v_0=0$¤

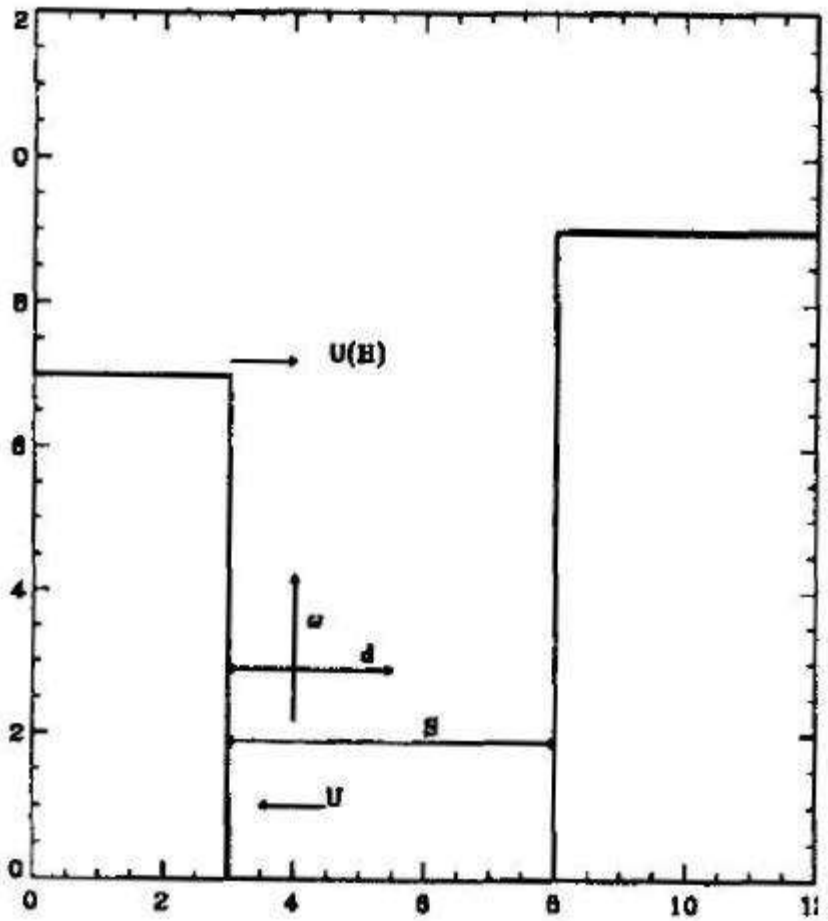
$$\frac{L_f}{H}=\frac{2(W/H)}{1+0.8W/H}$$

rear cavity and wake

¤	N¤	F¤
a_x,¤	¶ $L_r\sqrt{1-\left(\frac{z}{H}\right)^2}$ ¶ ¤	¶ $3L_r\sqrt{1-\left(\frac{z}{H}\right)^2}$ ¤
a_y,¤	$\frac{b_e}{2}$ ¶ ¤	$\frac{b_e}{2}$ ¤
composante·x¤	$u_0=-u(H)\left(1-\frac{d_l}{d_N}\right)^2$ ¶ ¤	$u_0=u(z)\left(1-\frac{d_N}{d_l}\right)^{1.5}$ ¤
composante·y¤	$v_0=-v(H)\left(1-\frac{d_l}{d_N}\right)^2$ ¶ ¤	$v_0=v(z)\left(1-\frac{d_N}{d_l}\right)^{1.5}$ ¤

$$\frac{L_r}{H}=\frac{1.8W/H}{(L/H)^{0.3}(1+0.24W/H)}$$

street canyon



si S < S**

$$\frac{S^{**}}{H} = \begin{cases} 1.25 + 0.15\,W/H & W/H < 2, \\ 1.55 & W/H \geqslant 2. \end{cases}$$

¶

$$U = -U(H)\frac{d}{0.5S}\left(\frac{S-d}{0.5S}\right) \cdot \cdot \cdot \cdot W = -\left|\frac{U(H)}{2}\left(1-\frac{d}{0.5S}\right)\right|\left(1-\frac{S-d}{0.5S}\right)$$ ¶

¶

S: street width

d: distance from grid point to upwind building

U(H) wind on roof of upwind building

for non perpendicular wind to canyon axis wind is decomposed in parallel and perpendicular components

- Buildings are defined
  - 4 corners, height
  - for industrial complex, take BPIP
- Each grid point is determined
  - free
  - inside a building
  - in zone: upfront, cavity, wake, canyon
  - search for street canyons is tedious
    - grid points in street canyons are saved in a file for further applications

Initial wind field

- MOST profile according to the weather conditions (wind, temperature, cloud ect) and local variables (roughness, albedo ect)
- Each grid point is attributed an initial wind field depending on its position with respect to building zones

Wind field solution

- Initial wind field is the start up wind for the application of a mass conservation model on the modelling domain (divergence minimization)

- the function E is minimize over the whole domain
- (u_0,v_0,w_0): initial wind field: wind attributed in various zones
- (u,v,w): final wind field

$$E(u,v.w)=\int_V\left[\alpha_1^2\left(u-u_0\right)^2+\alpha_2^2\left(v-v_0\right)^2+\alpha_3^2\left(w-w_0\right)^2\right]dV$$

- with a zero divergence constraint on the final wind field

$$\nabla \bullet V = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- is the same as to minimize J

$$J(u,v,w;\lambda)=\int_V \left[\begin{array}{l}\alpha_1^2\left(u-u_0\right)^2+\alpha_2^2\left(v-v_0\right)^2+\alpha_3^2\left(w-w_0\right)^2+\\ \lambda\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}+\dfrac{\partial w}{\partial z}\right)\end{array}\right]dV$$

- and λ(x,y,z) is subjected to the following identity and is solved numerically; R is called the source term (divergence)

$$\frac{\partial^2\lambda}{\partial x^2}+\frac{\partial^2\lambda}{\partial y^2}+\left(\frac{\alpha_1}{\alpha_2}\right)^2\frac{\partial^2\lambda}{\partial z^2}=R$$

then the final wind field (u,v,w) is obtained as a function of (x,y,z) with λ(x,y,z)

$$u = u_0 + \frac{1}{2\alpha_1^2}\frac{\partial \lambda}{\partial x}$$

$$v = v_0 + \frac{1}{2\alpha_1^2}\frac{\partial \lambda}{\partial y}$$

$$w = w_0 + \frac{1}{2\alpha_2^2}\frac{\partial \lambda}{\partial z}$$

- The λ equation is discretized as

$$R_{i,j,k} = \frac{\lambda_{i+1,j,k} - 2\lambda_{i,j,k} + \lambda_{i-1,j,k}}{\Delta x^2} + \frac{\lambda_{i,j+1,k} - 2\lambda_{i,j,k} + \lambda_{i,j-1,k}}{\Delta y^2} +$$

$$\left(\frac{\alpha_1}{\alpha_2}\right)^2 \frac{\lambda_{i,j,k+1} - 2\lambda_{i,j,k} + \lambda_{i,j,k-1}}{\Delta z^2}$$

$$R(I,J,K) = -2\alpha_1^2\, DIV(I,J,K)$$

- At solid surfaces such as wall and roofs the wind and the derivatives are null

$$\frac{\partial \lambda}{\partial x} = 0 \; ou \; \frac{\partial \lambda}{\partial y} = 0 \; ou \; \frac{\partial \lambda}{\partial z} = 0$$

- At points where there are solid surfaces discretized λ equation is adjusted to have zero derivatives. For example for a solid surface to EAST and one SOUTH

$$\frac{\partial^2 \lambda}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\right) = \frac{1}{\Delta x}\left(\left.\frac{\partial \lambda}{\partial x}\right|_{i+1/2} - \left.\frac{\partial \lambda}{\partial x}\right|_{i-1/2}\right)$$

$$= \frac{1}{\Delta x}\left(0 - \left.\frac{\partial \lambda}{\partial x}\right|_{i-1/2}\right) = \frac{1}{\Delta x}\left(-\left.\frac{\partial \lambda}{\partial x}\right|_{i-1/2}\right)$$

$$= \frac{\lambda_{i-1} - \lambda_i}{\Delta x^2}$$

$$\frac{\partial^2 \lambda}{\partial y^2} = \frac{\lambda_{j+1} - \lambda_j}{\Delta y^2}$$

- which is put back in the discretized equation

$$R_{i,j,k} = \frac{\lambda_{i-1,j,k} - \lambda_{i,j,k}}{\Delta x^2} + \frac{\lambda_{i,j+1,k} - \lambda_{i,j,k}}{\Delta y^2} + \left(\frac{\alpha_1}{\alpha_2}\right)^2 \frac{\lambda_{i,j,k+1} - 2\lambda_{i,j,k} + \lambda_{i,j,k-1}}{\Delta z^2}$$

- to obtain a value for $\lambda_{i,j,k}$

$$\lambda_{i-1,j,k} - \lambda_{i,j,k} + A\left(\lambda_{i,j+1,k} - \lambda_{i,j,k}\right) + B\left(\lambda_{i,j,k+1} - 2\lambda_{i,j,k} + \lambda_{i,j,k-1}\right) = \Delta x^2 R_{i,j,k}$$

$$\lambda_{i,j,k} = \frac{-\Delta x^2 R_{i,j,k} + \lambda_{i-1,j,k} + A\lambda_{i,j+1,k} + B\left(\lambda_{i,j,k+1} + \lambda_{i,j,k-1}\right)}{2\left(0.5 + 0.5A + B\right)}$$

$$A = \Delta x^2 / \Delta y^2 \quad B = \Delta x^2 \left(\alpha_1 / \alpha_2\right)^2$$

- Every point has its own equation depending on where is the solid surface (example wall to the NORTH, wall to WEST, roof UNDER)
- $\lambda_{i,j,k}$ field is then obtained iteratively according to the procedure given by Press (Numerical Recipes in FORTRAN)
- Final wind (u,v,w) is then obtained for all grid points
- Wind field for downtown Montréal (170 structures) calculated in 2 minutes: 1 min for initial search of canyon, 1 min for wind calculation, 4 millions grid points

AIRLAG MODEL

- Moves particules in the wind field (U,V,W) from AIRFLO
- Same spatial discretization
- Wind, buildings and other infos imported from AIRFLO output

Few equations

- Speed increments of a parcel moving in a wind field (U_1,U_2, U_3) are shown in Rodean, based on Thomson; these have a tensor form. The terms contain a deterministic part and a stochastic part to mimic turbulence

$$du_i = a_i(\vec{x},\vec{u},t)dt + b_{ij}(\vec{x},\vec{u},t)dW_j(t)$$

$$a_i = -\left(\frac{C_0\varepsilon}{2}\right)\lambda_{ik}(u_k - U_k) + U_j\frac{\partial U_i}{\partial x_j} + \frac{1}{2}\frac{\partial \tau_{ij}}{\partial x_j}$$

$$+\left[\frac{\partial U_i}{\partial x_j} + \frac{\lambda_{lj}}{2}\left(U_m\frac{\partial \tau_{il}}{\partial x_m}\right)\right]\left(u_j - U_j\right)$$

$$+\left[\frac{\lambda_{lj}}{2}\frac{\partial \tau_{il}}{\partial x_k}\right]\left(u_j - U_j\right)\left(u_k - U_k\right)$$

- expressing the tensors as summations

$$a_i = \sum_{k=1}^{3}\left(-\left(\frac{C_0\varepsilon}{2}\right)\lambda_{ik}(u_k - U_k)\right) \qquad T1$$

$$+\sum_{j=1}^{3} U_j \frac{\partial U_i}{\partial x_j} \qquad T2$$

$$+\sum_{j=1}^{3} \frac{1}{2}\frac{\partial \tau_{ij}}{\partial x_j} \qquad T3$$

$$+\sum_{j=1}^{3}\left[\frac{\partial U_i}{\partial x_j}\right]\left(u_j - U_j\right) \qquad T4a$$

$$+\sum_{l=1}^{3}\sum_{j=1}^{3}\sum_{m=1}^{3}\frac{\lambda_{lj}}{2}U_m\left(\frac{\partial \tau_{il}}{\partial x_m}\right)\left(u_j - U_j\right) \qquad T4b$$

$$+\sum_{l=1}^{3}\left(\frac{\lambda_{lj}}{2}\right)\sum_{j=1}^{3}\sum_{k=1}^{3}\left(\frac{\partial \tau_{il}}{\partial x_k}\right)\left(u_j - U_j\right)\left(u_k - U_k\right) \qquad T5$$

τ is the shear stress matrix

λ is inverse of τ

- and for a_1 !!!!

$$
\begin{aligned}
a_1 = & -\frac{C_0\varepsilon}{2}\left(\lambda_{11}\left(u_1-U_1\right)+\lambda_{12}\left(u_2-U_2\right)+\lambda_{13}\left(u_3-U_3\right)\right) \\
& +U_1\frac{\partial U_1}{\partial x_1}+U_2\frac{\partial U_1}{\partial x_2}+U_3\frac{\partial U_1}{\partial x_3}+\frac{1}{2}\left(\frac{\partial \tau_{11}}{\partial x_1}+\frac{\partial \tau_{12}}{\partial x_2}+\frac{\partial \tau_{13}}{\partial x_3}\right) \\
& +\frac{\partial U_1}{\partial x_1}\left(u_1-U_1\right)+\frac{\partial U_1}{\partial x_2}\left(u_2-U_2\right)+\frac{\partial U_1}{\partial x_3}\left(u_3-U_3\right) \\
& +\frac{1}{2}\left(U_1\frac{\partial \tau_{11}}{\partial x_1}+U_2\frac{\partial \tau_{11}}{\partial x_2}+U_3\frac{\partial \tau_{11}}{\partial x_3}\right)\left(\lambda_{11}\left(u_1-U_1\right)+\lambda_{12}\left(u_2-U_2\right)+\lambda_{13}\left(u_3-U_3\right)\right) \\
& +\frac{1}{2}\left(U_1\frac{\partial \tau_{12}}{\partial x_1}+U_2\frac{\partial \tau_{12}}{\partial x_2}+U_3\frac{\partial \tau_{12}}{\partial x_3}\right)\left(\lambda_{21}\left(u_1-U_1\right)+\lambda_{22}\left(u_2-U_2\right)+\lambda_{23}\left(u_3-U_3\right)\right) \\
& +\frac{1}{2}\left(U_1\frac{\partial \tau_{13}}{\partial x_1}+U_2\frac{\partial \tau_{13}}{\partial x_2}+U_3\frac{\partial \tau_{13}}{\partial x_3}\right)\left(\lambda_{31}\left(u_1-U_1\right)+\lambda_{32}\left(u_2-U_2\right)+\lambda_{33}\left(u_3-U_3\right)\right) \\
& +\frac{1}{2}\left(\left(\lambda_{11}\left(u_1-U_1\right)+\lambda_{12}\left(u_2-U_2\right)+\lambda_{13}\left(u_3-U_3\right)\right)\left(\frac{\partial \tau_{11}}{\partial x_1}\left(u_1-U_1\right)+\frac{\partial \tau_{11}}{\partial x_2}\left(u_2-U_2\right)+\frac{\partial \tau_{11}}{\partial x_3}\left(u_3-U_3\right)\right)\right. \\
& +\left(\lambda_{21}\left(u_1-U_1\right)+\lambda_{22}\left(u_2-U_2\right)+\lambda_{23}\left(u_3-U_3\right)\right)\left(\frac{\partial \tau_{12}}{\partial x_1}\left(u_1-U_1\right)+\frac{\partial \tau_{12}}{\partial x_2}\left(u_2-U_2\right)+\frac{\partial \tau_{12}}{\partial x_3}\left(u_3-U_3\right)\right) \\
& \left.+\left(\lambda_{31}\left(u_1-U_1\right)+\lambda_{32}\left(u_2-U_2\right)+\lambda_{33}\left(u_3-U_3\right)\right)\left(\frac{\partial \tau_{13}}{\partial x_1}\left(u_1-U_1\right)+\frac{\partial \tau_{13}}{\partial x_2}\left(u_2-U_2\right)+\frac{\partial \tau_{13}}{\partial x_3}\left(u_3-U_3\right)\right)\right)
\end{aligned}
$$

- Expressions are complex
- In a simple case without buildings one can use a reference system aligned with the mean wind i.e. with $U_2=0$, $U_3=0$ also (no vertical movement in the mean flow) and so many terms go to 0

- With buildings U_3 (vertical wind) may be non zero; but a moving doubly rotated system can have $U_2=0$ and $U_3=0$
- This was developped; but this requires continual change in reference frame following the particle and complex calculations (much time consuming) and interaction with buildings is difficult to follow
- Ordinary reference frame (x,y,z) is used

- To improve calculation speed all variables that could be computed before start are done (position dependent values are attributed to matrices)

Solid surfaces

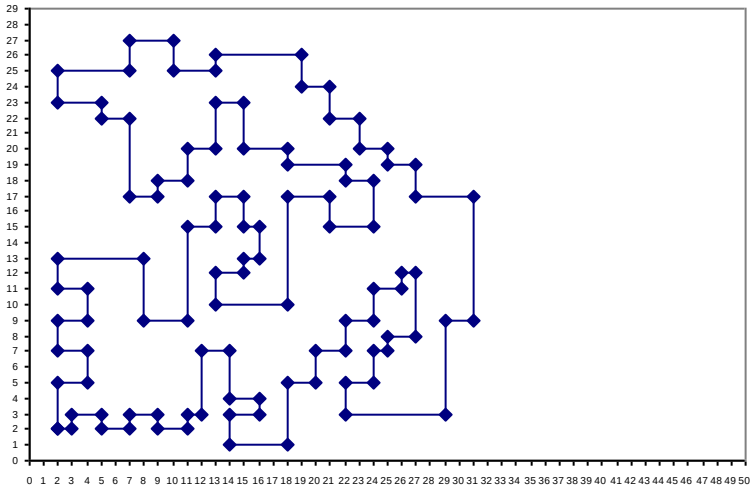
- Parcels are reflected on solid surface and on ground
- Tennis ball refection in 3d
- Special cases as ground to building, building corners, roof to wall ect are considered

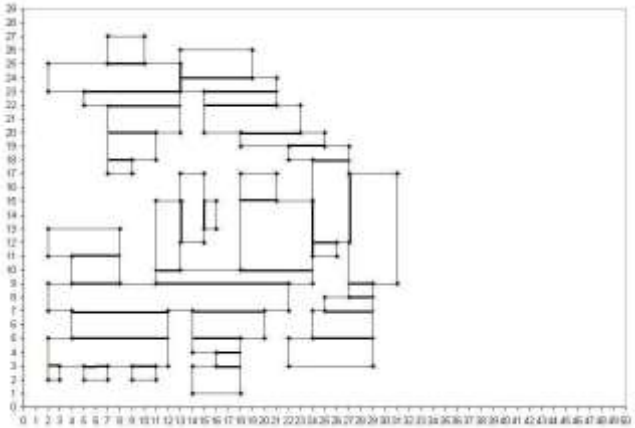
Examples

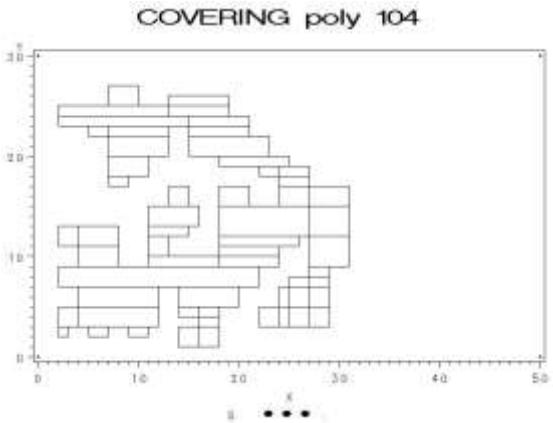
- Only qualitative results examples are shown here
- Model validation will be undertaken

- A short anecdote....☺!
  - Rockle parametrization is based on rectangular forms
  - non-rectangular buildings are thus approximated as superposition of rectangles
  - one would like to have some procedure to get rectangles from polygonal buildings; defined for example as in AERMOD VIEW with BPIP file

- efforts were devoted to program an algorithm to decompose concave rectilinear polygons in a minimum number of rectangles that superpose or do not superpose
  - what a job ☹☹.....
  - program will be made available on internet







original 104 vertex

juxtaposition

superposition

Québec, summer 2012: legionela episod
13 deads
origin: one cooling tower ; identified 20 september

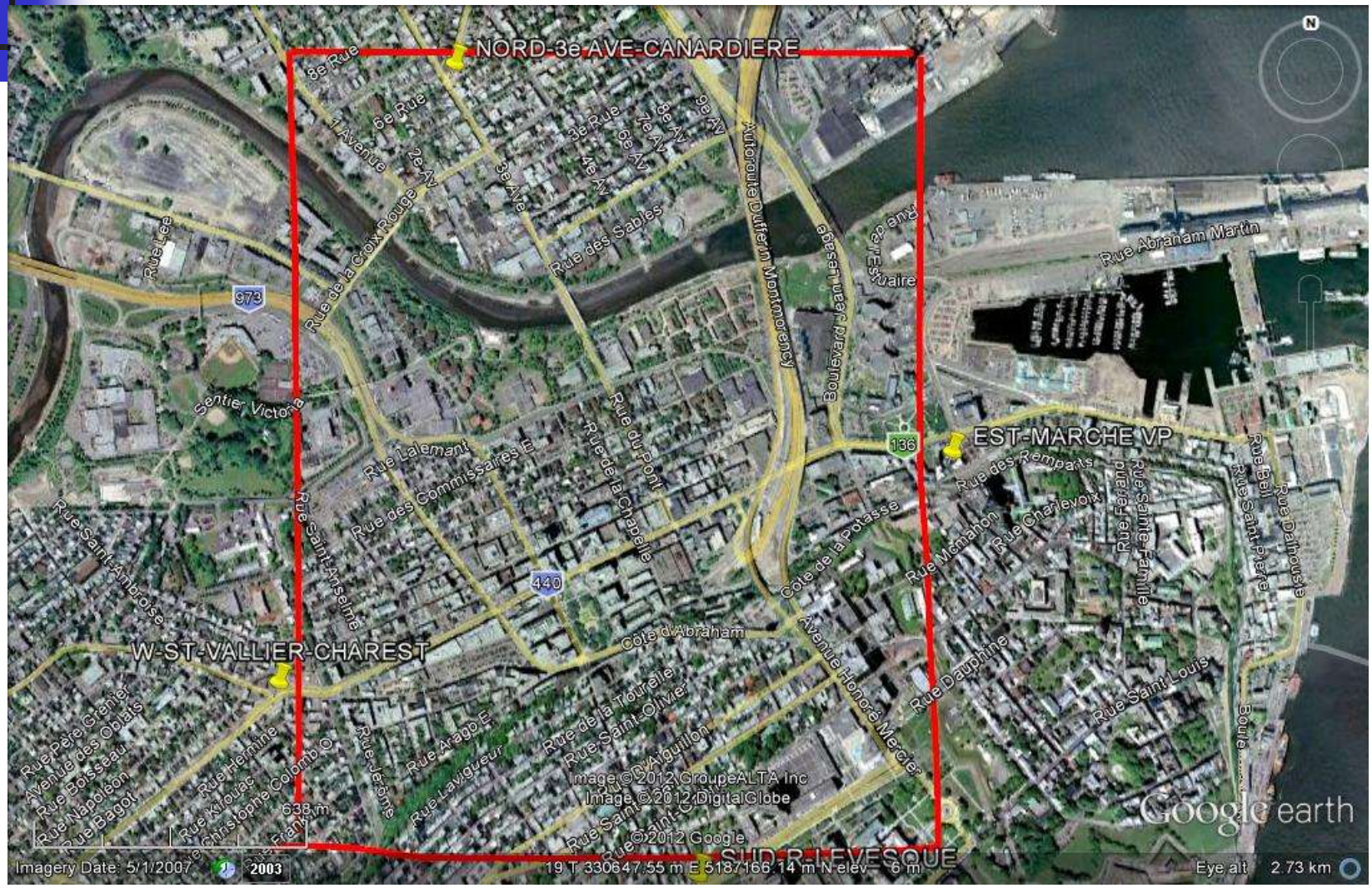
27/08/2012 news

a try for AIRLAG

as a volounteer test

the problem region were search was made

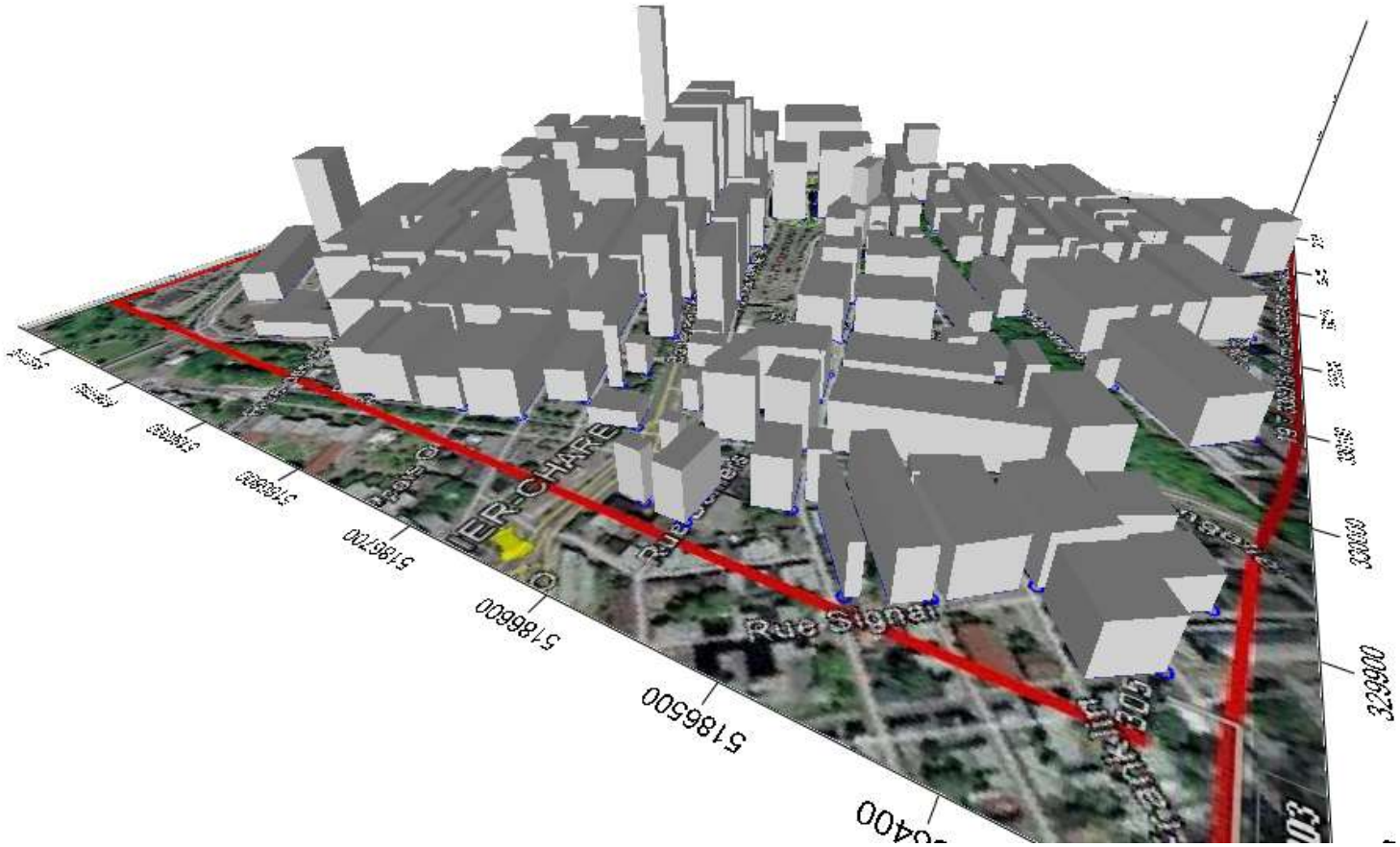
470 structures individual or joined (hand worked-no interface to municipal building data base yet) were input to AIRFLO/AIRLAG



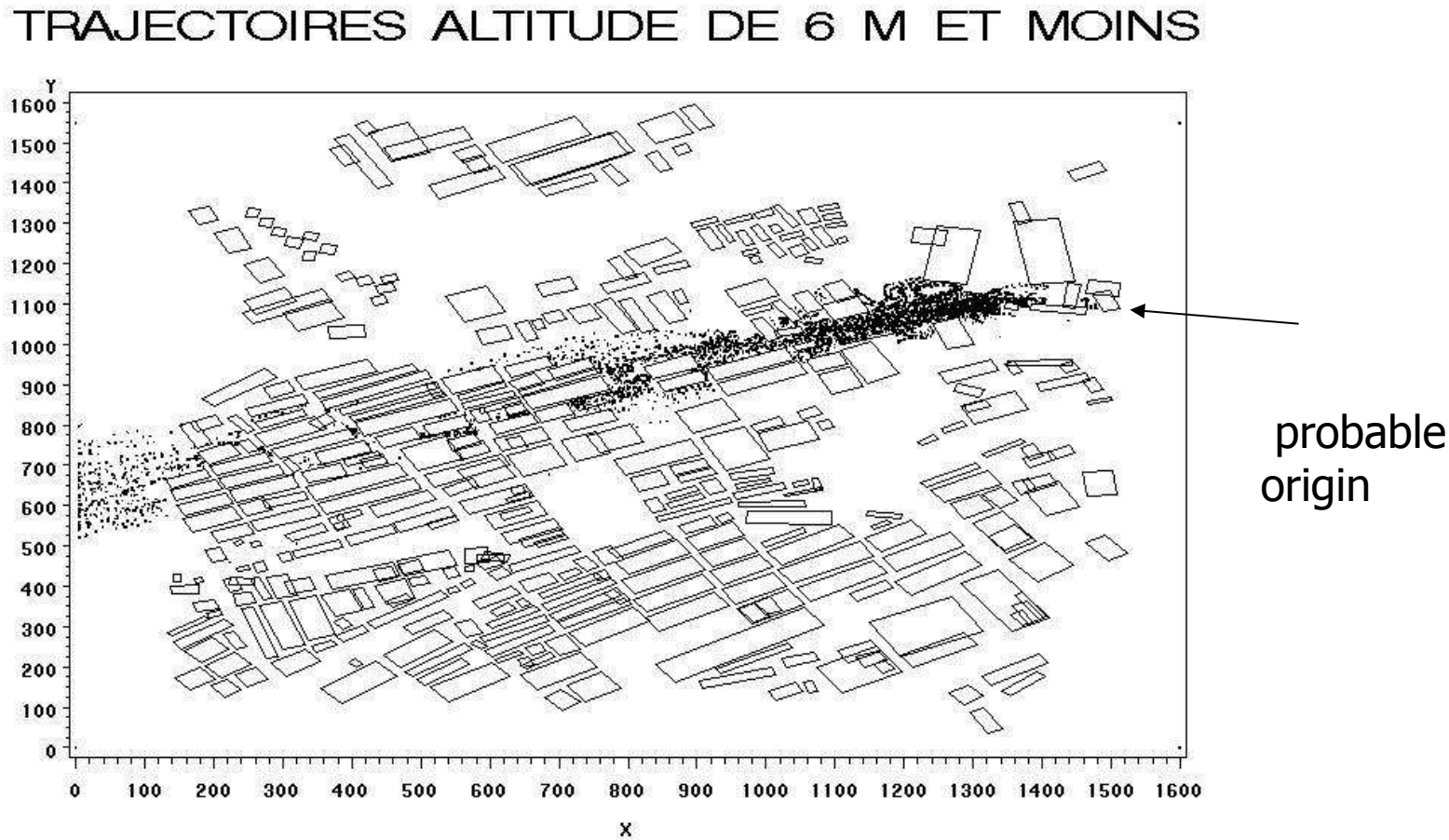
one part of the region, view towards NE

domain 1.5 km x 1.5 km

3D buildings view from AERMOD View



one place was suspected
trial: EAST wind, summer daytime

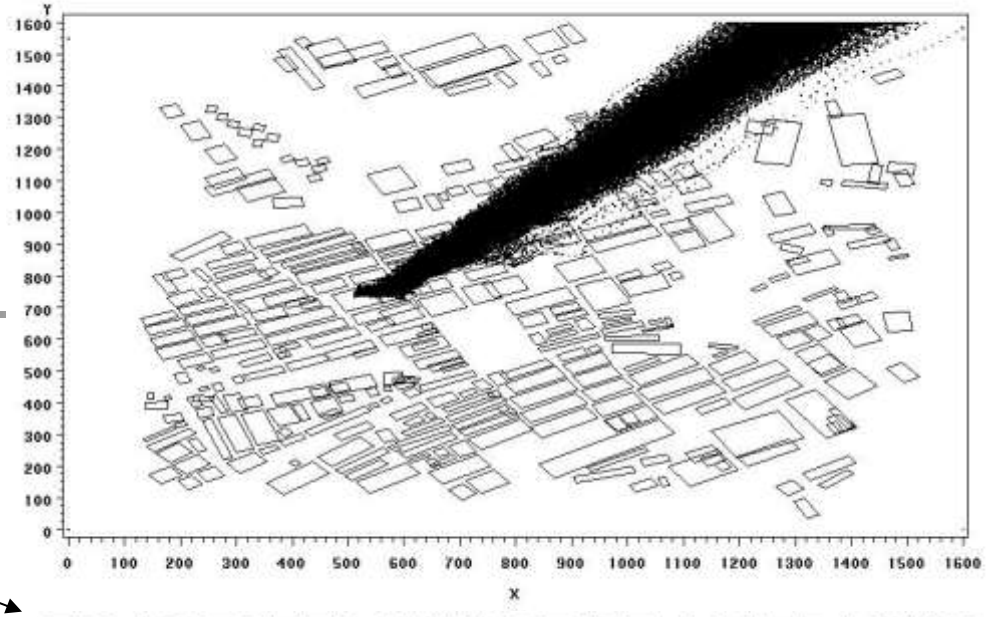


identified source roof

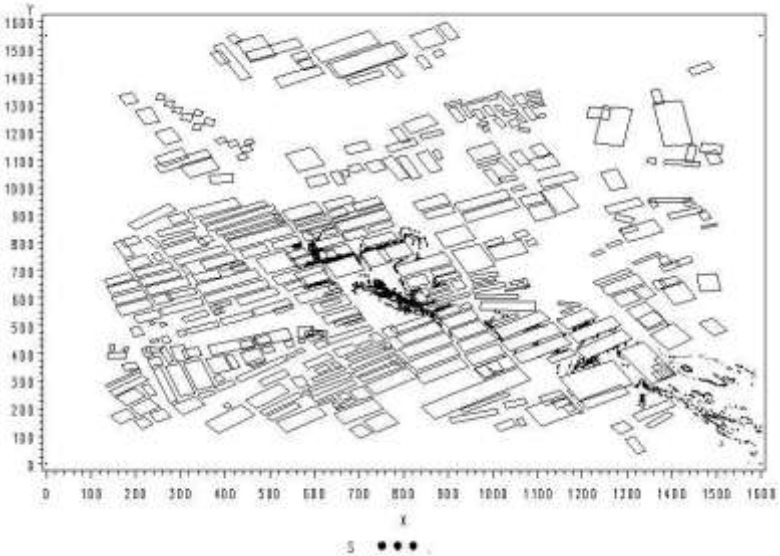
wind SW

wind NW

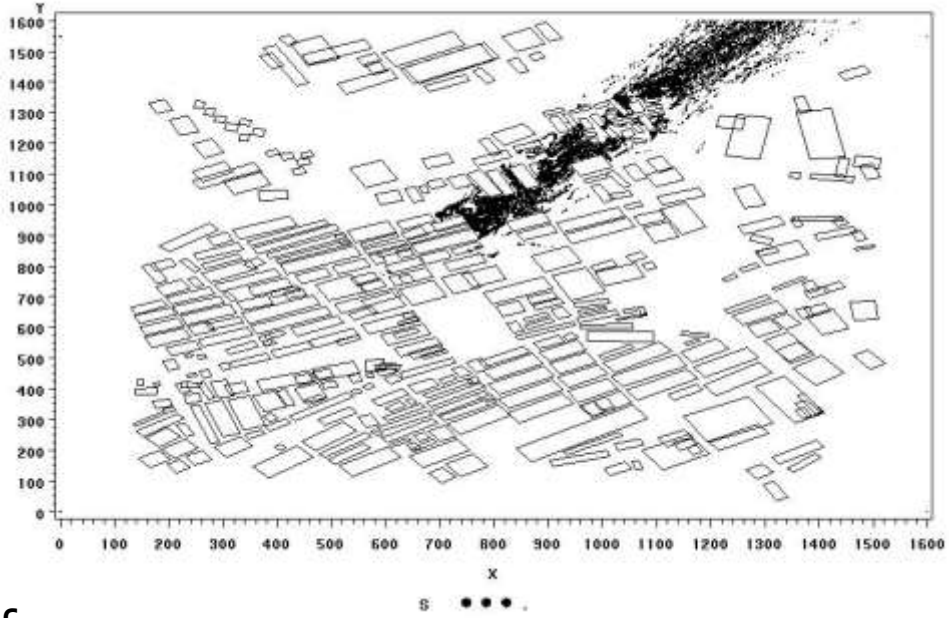
TRAJECTOIRES TOUTES ALTITUDES



TRAJECTOIRES ALTITUDE DE 4 M ET MOINS

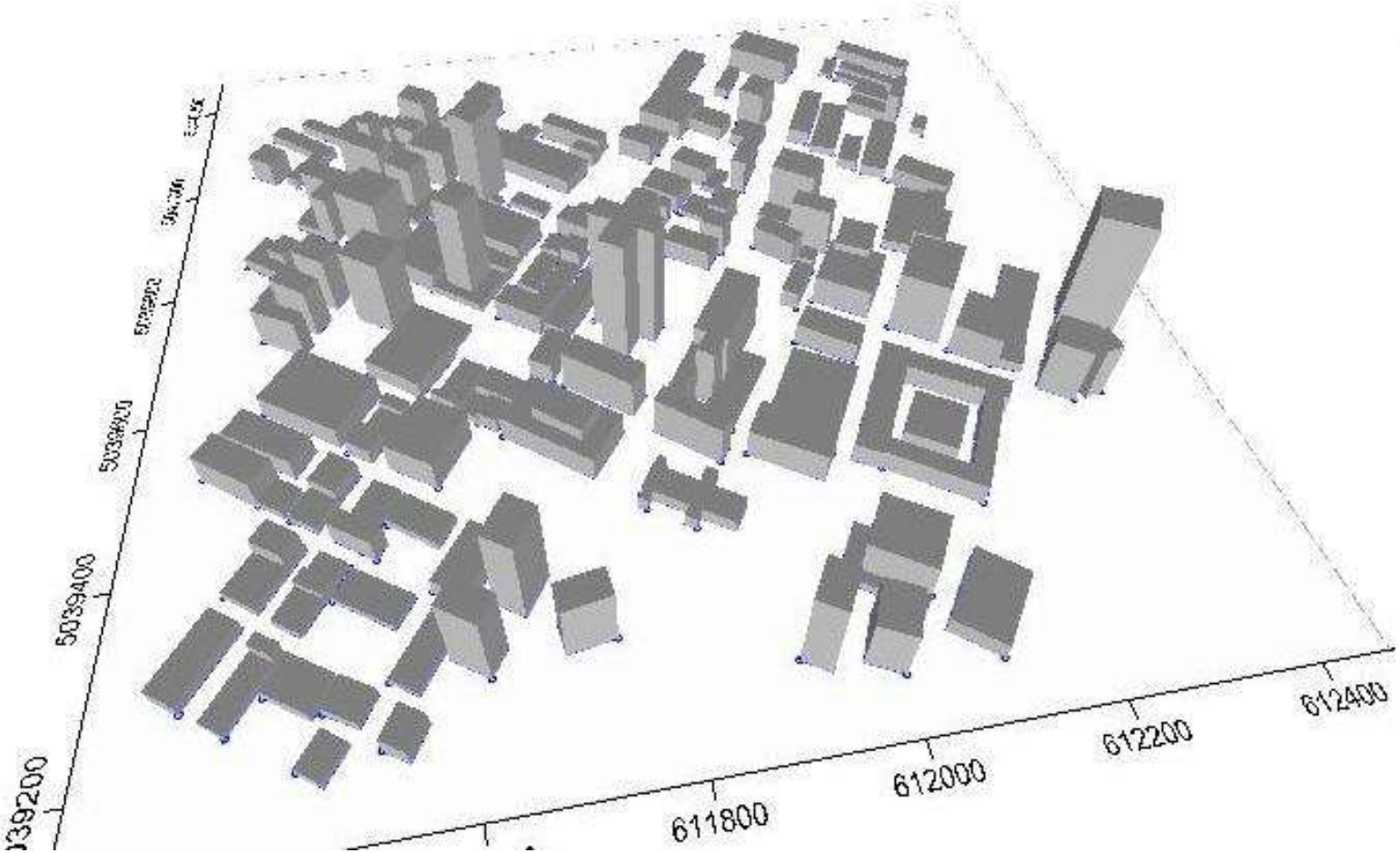


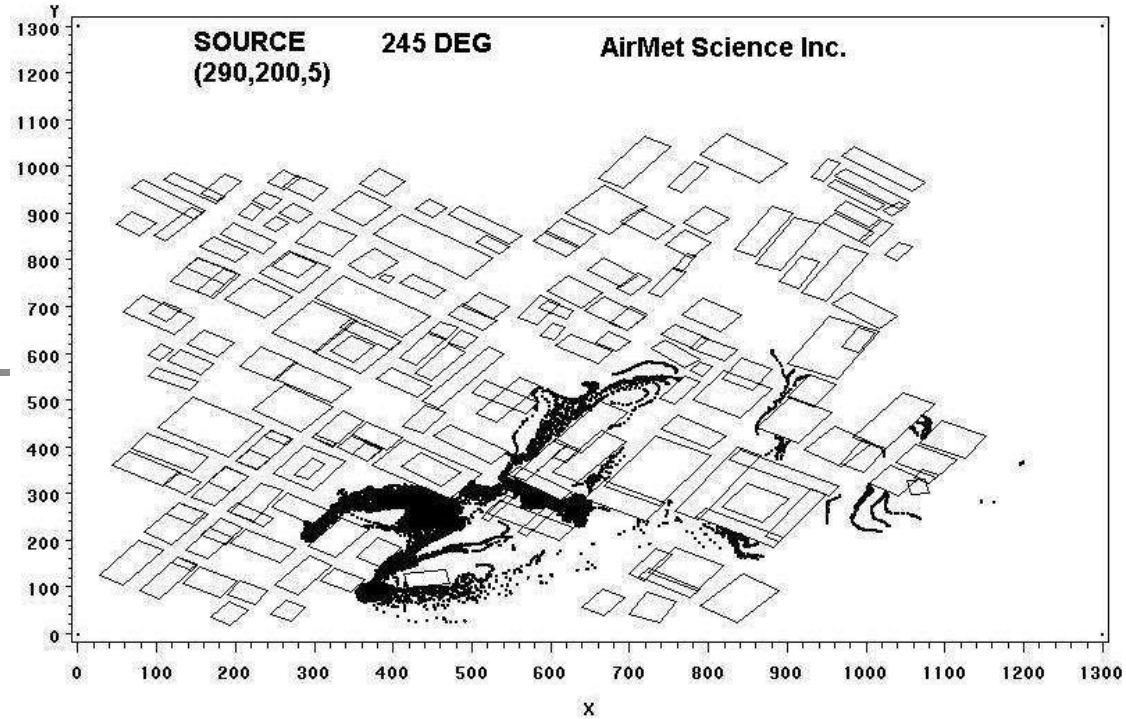
TRAJECTOIRES ALTITUDE DE 4 M ET MOINS

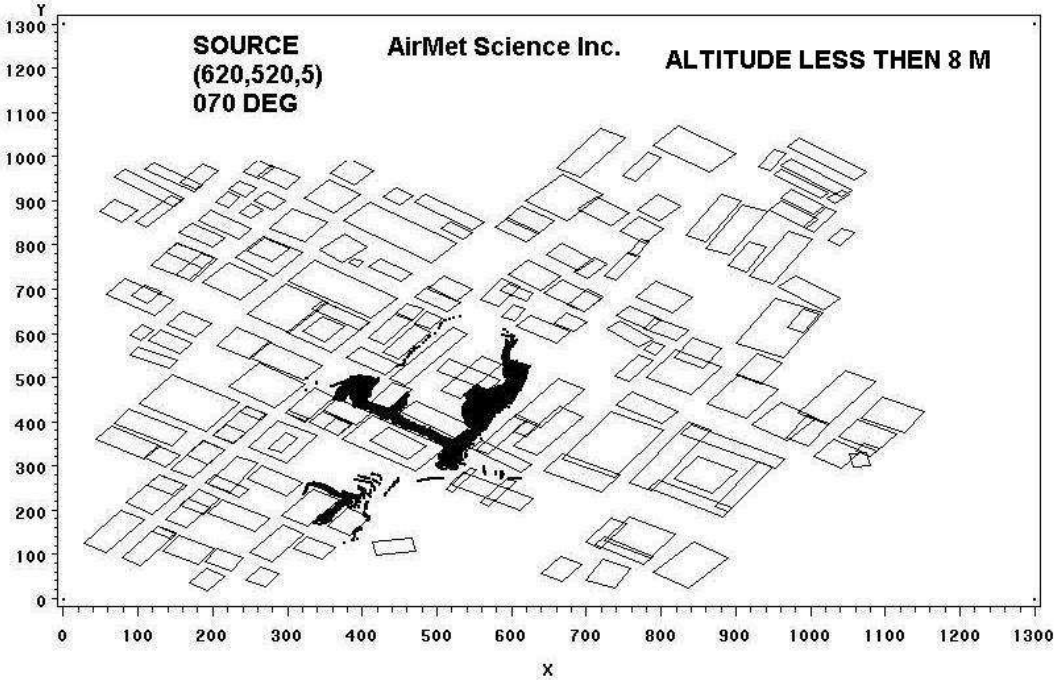


bacteria can reach people and go far

Montréal, part of downtown (170 structures) 3D from AERMOD View







complex circulation patterns

Ongoing and future works

- Vegetation effect
- Lagrangian fluctuations to calculate exceedances probabilities
- Topography
- Roof circulation
- Validation with wind tunnel experiments
- Improve code performance
- Migration to a better performing FORTRAN compiler
- Visual interface
- Wind field solution is still under questionning (CFD?)

Conclusion

- Development of this model (up to this point) required non negligeable efforts
- Further development appears interesting

THANKS

Lakes Environmental

olfactoexpert inc.

AirMet SCIENCE aer scientia